2.5 Continuity

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ALSO major milestone:

Intermediate Value Theorem (IVT):

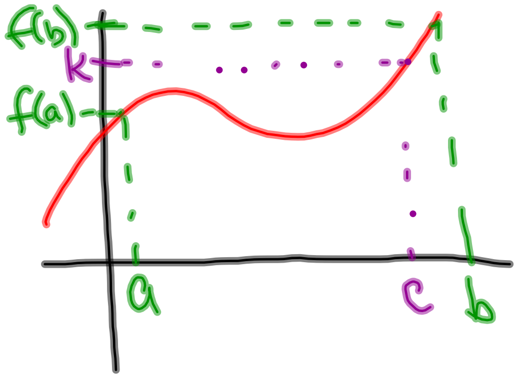
condition, hypothesis

If a function is continuous on $[a,b]$, and k is a number in $[f(a),f(b)]$ [or $[f(b),f(a)]$],

then there exists a c in $[a,b]$ with $f(c)=k$.

Conclusion

(Intermediate Value Theorem (IVT))



An existence theorem

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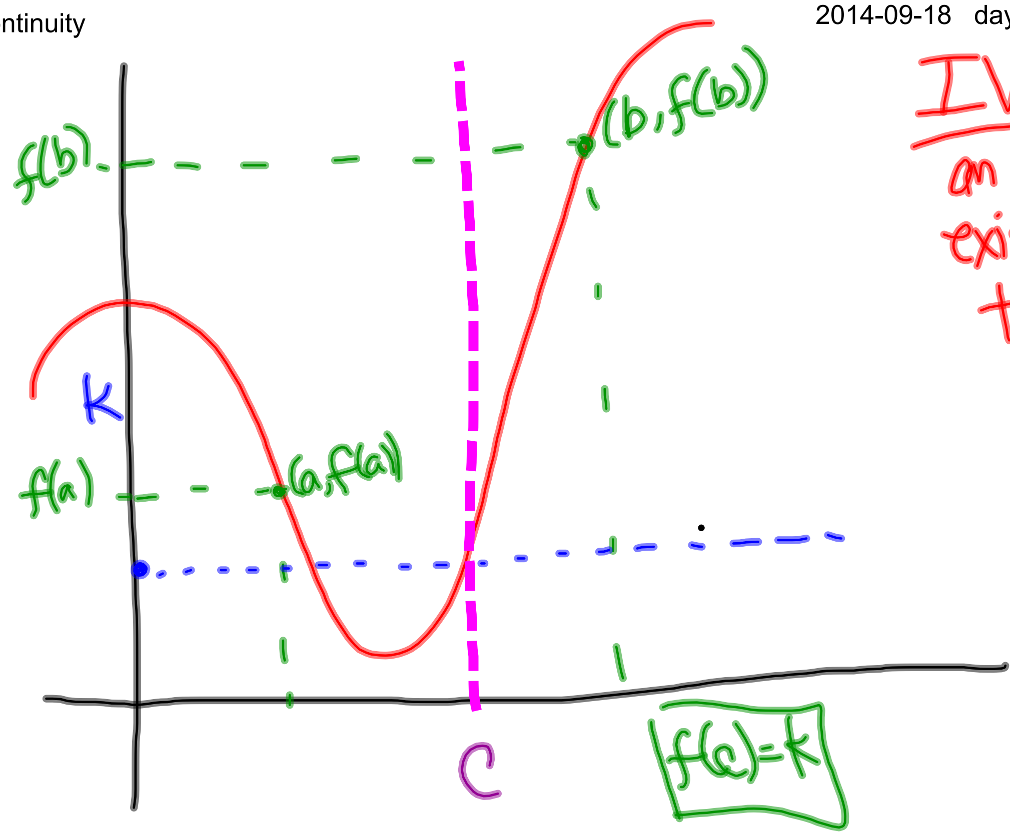

IVT

an existence theorem

2.5 Continuity Lemma - a fact or "little theorem" 2014-09-18 day 17

Theorem 1

Theorem 2

$\Downarrow$

Corollary

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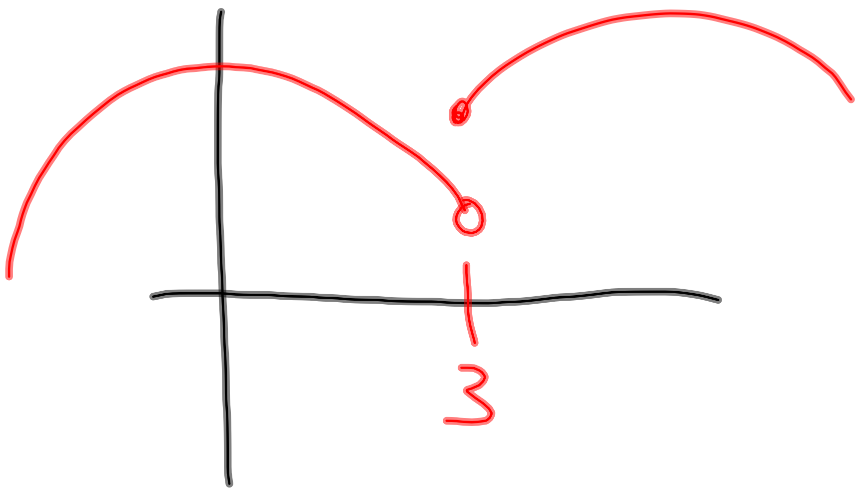
9) a) f is continuous everywhere except $x=3$, at which pt it is cont. from right



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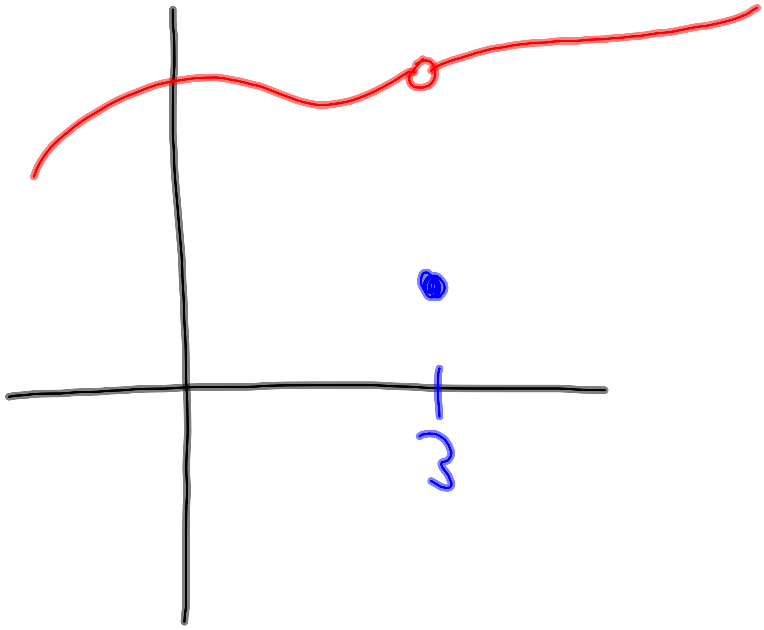
9b) 2 sided lim @ x=3; but not cont.



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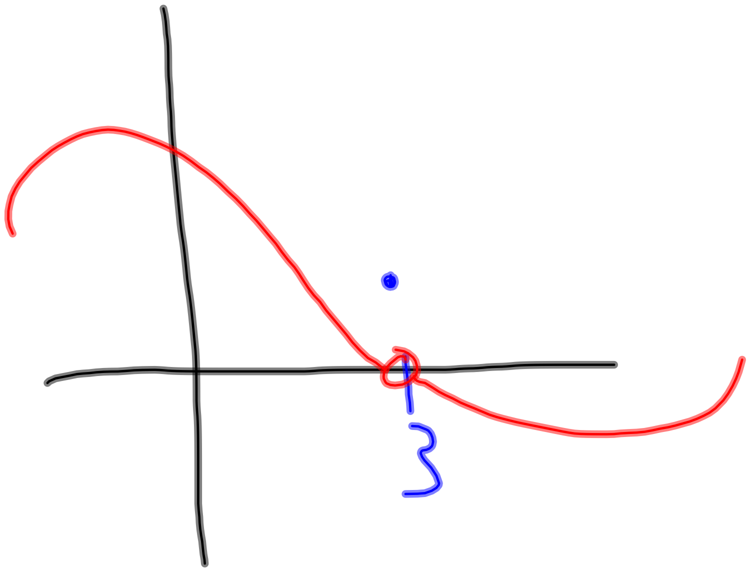
9 sea)

f not cont. @ $x=3$, $f(3)=1$; if $f(3)=0$ then it would be continuous



2.5 Continuity

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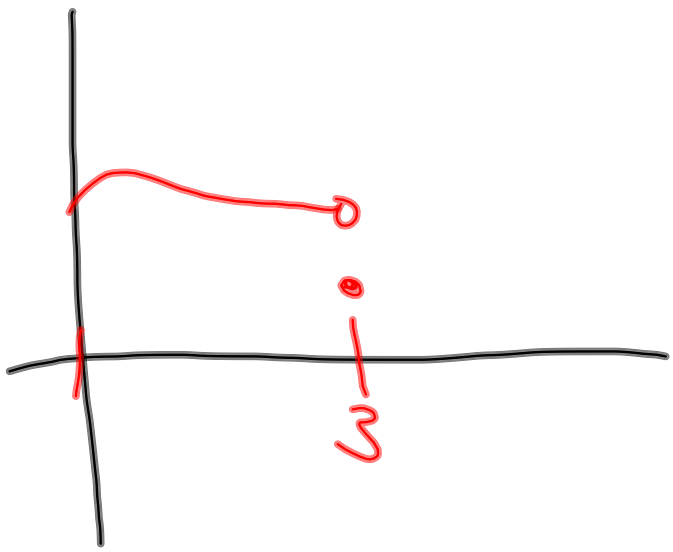
9d) f is cont on [0,3), and is def^{n} on [0,3], but f is not cont on [0,3]



2.5 Continuity

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7) f, g cont. $f(2)=1$;

$\bigstar \lim_{x \to 2} [f(x) + 4g(x)] = 13.$

1) $\bigstar = \lim_{x \to 2} f(x) + 4 \lim_{x \to 2} g(x)$.

[thm. b/c f, g cont the limits exist]

2) $= 1 + 4 \lim_{x \to 2} g(x) = 13$

[f is cont and $f(2)=1$]

3) $4 \lim_{x \to 2} g(x) = 12$; so $\underline{\lim_{x \to 2} g(x) = 3}$

so $\underline{g(2)=3}$

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13) $f(x) = x^3 - 2x + 3$

"Polynomials are continuous everywhere"

suppose we have an x-value where $f(x)$ is not continuous; say $x=a$

1) $f(a)$ would be a^3-2a+3, which is a #.
so $f(x)$ is defined at $x=a$ ✓

$\lim_{x\to a} 3$

2) $\lim_{x\to a} x^3-2x+3 = \lim_{x\to a} x^3 - 2\lim_{x\to a} x + 3$

$\lim_{x\to a} x^n = a^n$

$= a^3-2a+3$ which is a #

and so, two sided limit exists ✓

3) But wait! a^3-2a+3 [from 1] $= a^3-2a+3$ [fr 2

Sacré Bleu!

so, $f(x)$ is continuous at $x=a$.
which is a contradiction.

2.5 Continuity

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10) find formulas that are continuous on the intervals $(-\infty,0)$ and $(0,+\infty)$

but not cont on $(-\infty,\infty)$

$$f(x) = x \cdot \frac{\cancel{(x-2)}}{(x-2)}$$

$$f(x) = \frac{x}{x}(x-2)$$

2.5 Continuity

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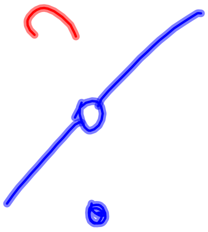
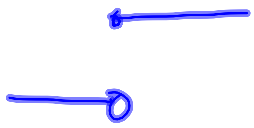
continuous



removable discontinuity

jump discontinuity



infinite discontinuity Vertical asymptotes.